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NUMBER AS A FORM-BUILDING PRINCIPLE OF MUSICAL COMPOSITION

Objective. The research is devoted to some aspects of the interaction of music and mathematics, namely, compositional designs based on fundamental world constants (such as the Pi number and Euler number) or statistical data embodied in some way by the authors in the sound fabric of their works. In addition to the musicians' dialogue with mathematics, the article gives an example of a reverse dialogue (Leonard Euler's mathematical research in the field of music) and discusses the issue of dissonance in music. **The methodology** consists in the use of comparative, analytical, historical-logical methods. **The scientific novelty** of the work is to expand the prevailing notions of the concomitant role of the mathematical component in the process of composer creativity to the level when it becomes the formative primary principle. **Conclusions.** The object of the mathematical world, becoming the starting ideological point of musical composition and penetrating into the structure of a musical fabric, transforms it according to its needs. Transplanted into the sound sphere, this object generates a series of already completely musical thoughts of the author, however, his symbolic fundamental principle continues to remind of himself throughout the entire musical development.

Keywords: Pi number, Euler number, musical composition, musical form, dissonance semantics.

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Число як формоутворюючий принцип музичної композиції.

Мета роботи. Дослідження присвячене деяким аспектам взаємодії музики і математики, а саме - композиторським задумам, основу яких складають фундаментальні світові константи (такі як число Пі і число Ейлера) або статистичні дані, втілені авторами деяким чином в звуковій тканині їх творів. Крім діалогу музикантів з математикою, в статті наводиться приклад зворотного діалогу (математичні дослідження Леонарда Ейлера в області музики) і розглядається питання, що виникає в зв'язку з цим, про дисонанс в музиці. **Методологія** полягає в використанні компаративного, аналітичного, історико-логічного методів. **Науковою новизною** роботи є розширення існуючих уявлень про супутню роль математичної складової в процесі композиторської творчості до рівня, коли вона стає формотворною першоосновою. **Висновки.** Об'єкт математичного світу, стаючи відправною ідейної точкою музичної композиції і проникаючи в структуру музичної тканини, трансформує її відповідно до своїх потреб. Трансплантуючись до звукової сфери, цей об'єкт породжує низку вже цілком музичних думок автора, проте його символічна першооснова продовжує нагадувати про себе протягом усього музичного розвитку.

Ключові слова: число Пі, число Ейлера, музична композиція, музичне формоутворення, семантика дисонансу.

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Число как формообразующий принцип музыкальной композиции

Цель работы. Исследование посвящено некоторым аспектам взаимодействия музыки и математики, а именно – композиторским замыслам, основу которых составляют фундаментальные мировые константы (такие как число Пи и число Эйлера) либо статистические данные, воплощенные авторами некоторым образом в звуковой ткани их произведений. Помимо диалога музыкантов с математикой, в статье приводится пример обратного диалога (математические исследования Леонарда Эйлера в области музыки) и рассматривается возникающий в связи с этим вопрос о диссонансе в музыке. **Методология** состоит в использовании компаративного, аналитического, историко-логического методов**. Научная новизна** работы заключается в расширении бытующих представлений о сопутствующей роли математической составляющей в процессе композиторского творчества до уровня, когда она становится формообразующей первоосновой. **Выводы.** Объект математического мира, становясь отправной идейной точкой музыкальной композиции и внедряясь в структуру музыкальной ткани, трансформирует ее сообразно со своими потребностями. Трансплантируясь в звуковую сферу, этот объект порождает череду уже всецело музыкальных мыслей автора, однако его символическая первооснова продолжает напоминать о себе на протяжении всего музыкального развития.

Ключевые слова: число Пи, число Эйлера, музыкальная композиция, музыкальное формообразование, семантика диссонанса.

Relevancy. XX century brought many innovative ideas and interdisciplinary interrelations into musical art. One of the key ones is cooperation between music and mathematics that gave birth to both penetration of mathematical principles into musical and composing process, and a whole line of mathematical researches in musical-theoretical area. Interest to similar dialogues continues to exist and nowadays, but at some another level.

Purpose of work – to study some aspects of cooperation of music and mathematics and modern stage, to show that mathematical object can be not only indirect participant of musical process, but also the main shaping element of the composition.

Scientific novelty of the work lies within widening the present concept of accompanying role of mathematical component in the process of composer's creativity to the level when it becomes shaping original basis.

Statement of main content. The number is one of the fundamental concepts in shaping the world outlook of mankind. Since ancient times, people are trying to penetrate into its abstract essence, viewing it under different disciplinary angles. Thus, mathematicians pay special attention to the properties and principles of arrangement of logical-numerical and conceptual constructions, representatives of natural science fields actively apply the practical results of their research, philosophers undertake the search for general cosmological laws and touch on ontological questions in connection with the concept of number [2].

Despite the numerous studies in this direction The question of the role of numbers in music remains relevant to this day. A. F. Losev writes: "Music arises as the art of time, in the depths of which (time) lies the perfectly immovable figure of a number and which outwardly blossoms with the qualities of the materialized movement" [3, 545].

In the history of music, there are many examples of the synthesis of music and mathematics, visually or invisibly in music there are various proportions (a mandatory parameter, for example, is the point of the golden section in compositional architectonics), some composers knowingly use numerical sequences (for example, the Fibonacci series and the Lucas series[1] [1]). Interest in identification of mathematical regularities in music, intuitively or intentionally laid by the authors in their creations, has existed in musicology for a long time, transforms with the development of musical and theoretical thought and is gradually growing as the achievements of scientific and technological progress are

[1] Thus, the form of one of the famous compositions by S. Gubaidulina "In the Beginning There War Rhythm" for an percussion instruments ensemble (1984) defines the Fibonacci numbers. The Fibonacci series is also used in rhythmic organization of some compositions of Stravinsky, Bartok, Stockhausen. Lucas series is represented in the compositions by Debussy, Gubaidulina.

introduced into our lives. In this respect the twentieth century has become a real chest of musical ideas, the echoes of which continue to live in the present 21st century. It is interesting to observe the process of their change, being overgrow with new meanings and forms arising from the needs of the modern era. In the twentieth century, various compositional techniques (serial, serial, pointillism, sonorism, aleatory music, electronic music, etc.), in which mathematics participates mainly indirectly (in a structural framework, the algorithmic principle of the structure of a composition, the technical and acoustic factor), in the 21st century the idea of direct dialogue with mathematics became popular.

As an example of such a dialogue, one can cite a musical composition for violoncello, written by a student of the University of Minnesota, Daniel Crawford, called “A Song of Our Warming Planet” (2013). Using the annual average temperature of the Earth, collected by the Goddard Institute from 1880 to 2012, he built an appropriate data graph and reflected them in his composition by means of sounds (low notes represent relatively cool years, while high notes are warm). The temperature data are presented in the range of three octaves, beginning with the coldest mark in the entire history of observations (-0.47 ° C in 1909), which is indicated by the lowest note. Each rise per semitone is approximately 0.03 °C of global warming. In this way, the author tried to bring the understanding of climate issues to a wide audience. According to Crawford, the melody can explain it more intelligibly than the charts and figures.

The famous Pi number ($\pi \approx 3{,}14159265358979323846264 3 \ldots$) didn’t remain without attention. Being irrational and, moreover, transcendent, this number demonstrates its comprehensive universality and does not cease to amaze humanity with its mysterious ability to unexpectedly show up in the most diverse sections of mathematics, showing an interrelation with concepts very distant from it.

The Pi number holds special place in art, monuments[2] are established to honor it, it is used in proportions of architectural constructions[3], they are dedicated to stories[4] and films[5].

[2] In Seattle, on the steps in front of the arts museum building; in USA, Sculpture Park (New Jersey); on the southern coast of the Crimean Peninsula close to the Katsiveli City.

[3] German King Friedrich II was so charmed with the magical number π, that he dedicated the whole Castle del Monte to it, in proportions of which this magical number is preserved.

[4] In 1996 Mike Kate dedicated a short story “Rhythmical Cadence”. Length of the words in texts equals the first 3834 numbers of π.

[5] In 1998 the Director Darren Arnovski filmed the psychological thriller “Pi”; in 2012 the dramatic film “Life of Pi” directed by Ang Lee saw screens.

These facts indicate that interest in Pi number went far beyond the limits of mathematical circles.

The famous figure attracted the attention of musicians. Embodying in a sequence of numbers of numbers of Pi in various ways, they received rather interesting results.

David Macdonald played 122 digits after the decimal point of π, assigning each digit a corresponding step in A minor and harmonizing the received melody in an arpeggiated texture on the piano in a rather traditional and unsophisticated way. His video “Song from π!” got almost 5 million views in Youtube and for many (judging by the comments) became a true revelation in the world of music.

Another example is “The Melody of Pi”. Its author, Jim Zamerski, used π in the 12-digit system of calculus and put the sound of the 12-step chromatic scale from “C” (supplemented it with “zero” - the sound B, that precedes the initial single tone) for each digit. Its version involves 226 characters of writing the number π. In this case, the number itself again becomes a melodic frame, on which harmony is imposed, this time described by a waltz texture on the piano (other electronic timbres appear only in the culmination, thus supporting the melodic and harmonic layers of the composition). Due to the use of the 12-step basis, this composition is more interesting from the harmonic side and from the drama.

The third example is again a call to diatonic, but this time from the sound "to" (which corresponds to the digit 1), in order, with the inclusion of "before" and "re" of the second octave (8 and 9). In its composition "What Pi sounds like" its author, Michael John Blake, in addition to matching the figures of individual sounds, uses and matching them with a triad (built from the corresponding steps in C major). This composition uses only 32 signs of π, but it is musically the most interesting of those considered due to the timbre variety and dramatic crocheting with the gradual connection of instruments. The original theme, sounding monotonous, is fixed as an ostinat background and is colored with various bell-shaped polyphonic combinations with oneself, reaching the end before the climax of apotheosis. By successively combining instrumental timbres that are different but subordinate to the solution of one numerical problem, the effect of "world harmony" is created as a reflection of the universal essence of π.

In addition to the well-known number Pi, there is in mathematics an equally fundamental and omnipresent number that permeates all the branches of mathematics (which in the school course is mentioned in connection with the notion of the natural logarithm and exponential function) is the Euler number ($e \approx 2{,}71828182845904523 \ldots$). Like the number π, it is irrational and transcendental.

Leonard Euler (1707-1783), whose name is associated with the number "e", the person is legendary in mathematics. The Swiss, German and Russian mathematician and mechanic, who made a fundamental contribution to the development of these sciences (as well as physics, astronomy and a number of applied sciences), author of more than 850 works (including two dozens of fundamental monographs) on mathematical analysis, differential geometry, number theory, approximate calculations, celestial mechanics, mathematical physics, optics, ballistics, shipbuilding, music theory and other fields. He deeply studied medicine, chemistry, botany, aeronautics, music theory, many European and ancient languages. Academician of the Petersburg, Berlin, Turin, Lisbon and Basel Academies of Sciences, a foreign member of the Paris Academy of Sciences.

Well known in natural-mathematical circles, he is rarely mentioned in musical literature, despite his very significant contribution to the field of musical theory. (Evil tongues rumored that in his works on music “there was too much music for mathematicians and too much mathematics for musicians.”) Euler was interested in musical harmony throughout his life, trying to give her a clear mathematical justification.

The purpose of his early work – “Experience of the New Music Theory" (Tentamen novae theoriae musicae, 1739) – was an attempt to describe mathematically how pleasant (harmonious) music differs from unpleasant (dissonant) [4].

Taking for the basis of the overtone scale, it relates the degree of sonority to the degree of simplicity of the rational relations of the frequencies of sounds (that is, their “overtone numbers” in the scale). Euler introduces a value characterizing the degree of pleasure – GS (gradus suavitatis).

He accepts GS(1)=1, GS(p)=p (for simple numbers). Then he receives the mathematical equivalent to the point that the intervals preserve the degree of their sonority regardless of the height of their initial tone:

$$GS(mn)-GS(n)=GS(m)-GS(1), \text{ т.е.}$$

$$GS(mn)=GS(m)+GS(n)-1. \qquad (1)$$

The consequence of that is the correlations:

$$GS(2n)=GS(n)+GS(2)-1=GS(n)+1, \qquad (2)$$

$$GS(p_1 p_2 \ldots p_k)= p_1+ p_2+\ldots+ p_k-(k-1). \qquad (3)$$

Now one can easily calculate the “grade of sonority” of any interval. Let us take the correlation of frequencies of sounds m:n (the received decimal must be simplified, in other cases transform it to that state), decompose the product mn to simple multipliers p_1 p_2… p_k and calculate

$$GS(m:n)=GS(mn)= GS(p_1 p_2\dots p_k)$$ using the formula (3).

For example, for small tenth (5:12) we shall receive:

$$GS(5:12)=GS(5*12)=GS(2*2*3*5)=2+2+3+5–3=9.$$

For comparison, small third (5:6) shall result in:

$$GS(5:6)=GS(5*6)=GS(2*3*5)=2+3+5–2=8.$$

That is, third sounds less sharply than third.

Similarly, one could note that the octave (1:2) (for which GS(1:2)=2) is more sonorous than the interval equal two octaves (GS(1:4)=GS(4)=GS(2)+1=3).

It is interesting that duodecimo (1:3) and two octaves interval (1:4) turn out to be equally “pleasant”: GS(3)=3, GS(4)=GS(2)+1=2+1=3.

Also, small sixth (5:8) and large second (8:9) are equally pleasant (natural intervals are meant). Really,

$$GS(5:8)=GS(5*8)=GS(2*2*2*5)=2+2+2+5–3=8,$$

$$GS(8:9)=GS(8*9)=GS(2*2*2*3*3)=2+2+2+3+3–4=8.$$

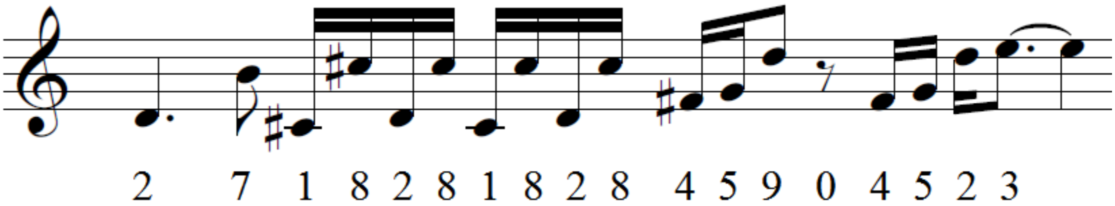
This approach in general corresponds to our auditory sensations.

At the end of Chapter VII of the “Experience” Euler arranged the intervals under the “degrees of pleasure” (gradus suavitatis), while the octave was assigned to the II (most pleasant) class, and the diaschisma to the last, XXVII class (the most discordant interval).

In the decline of years, in 1773, Euler read a report at St. Petersburg Academy of Sciences, in which he finally formulated his latticed representation of the sound system, metaphorically called by the author as a “mirror of music” (Latin speculum musicae). Euler's report was published in the form of a small treatise De harmoniae veris principiis per speculum musicum repraesentatis (“On the true foundations of harmony represented through speculum musicae”). Under the name of the “sound network” (German Tonnetz), the Eulerian grid was widely used in the German musical theory of the 19th century.

Unfortunately, the number of Euler in music communities is almost unknown and until recently did not inspire composers for creative searches. The author of this article was able to eliminate this “injustice” in the work for the flute, oboe / English horn, violin, cello and piano “Harmony of Euler’s number”[6] (2016).

The basis of the work was a theme, each sound of which corresponds to the figure of the Euler number (1 – C, 2 – D, 3 – E, ..., 8 - again C, 9 - D, 0 - pause).



However, some “error”, “defect” (“C-sharp” and “F-sharp” instead of "before" and "fa") crept into the original version of the theme (embodying the beauty and harmony of the universe), that “Pythagorean coma”, which seeks to destroy the delicate balance of the world. This “flaw” becomes a “worm” that undermines the world from within and leads to disaster. The world is collapsing, only the fragments of the past are left. But soon something begins to form from these fragments, gradually taking shape – and the theme is reborn. The equilibrium is restored, the universe regains its beauty and harmony, but this harmony is already different (“you cannot enter the same river twice”). It would seem that you can breathe a sigh of relief – but no, the "flaw" has not disappeared...

The work has a three-part structure. In the first part after a short introduction (in which, along with harmony, there is also a "flaw"), the main character appears – the theme of Euler's number at the violin. Then the variants of the theme sound in turn using the flute, cello and English horn, rapidly gaining momentum and leading to the culmination of its conduct at the violin against the background of the piano's ostinato pulsation and the soaring replicas of the remaining instruments.

After the “bell-like” destructive episode comes the second section, built on the wreckage of the theme and the “defect” reminding of itself. In this section, a great role is played by the percussion component of the musical fabric, created as instrumental tools (use of a rain staff, flute tapping, playing tremolo behind the stand and glissading replicas of string instruments, various piano sounds – contrary double surface glissading with pencils on piano keys, pencil drop sound on piano strings in different registers, etc.), and with the help of a phonetic verbal effect (the sound of "Pf" with a clear accented attack and smooth fading to complete extinction, which symbolizes the “breath of the world”). The latter effect arises

[6] «Harmony of Euler’s Number»

and actively manifests itself in the first section, then disappears for some time in the episode of “ruins”, returning again only by the end of this section and signifying the restoration of the disturbed harmonic balance, the world reviving from the ruins. The theme, scattered in this section on the set of motives that live with each one's life, gradually acquires ever greater wholeness and clarity of outlines, and at the beginning of the third section is going to be put together.

The first conduct of the “restored theme” at the beginning of the third section involves almost all performers. Individual replicas of the theme consistently pass through various instruments, dying at the last sound of their fragment and transferring the “baton” to its next participant, gathering gradually into a single chord-cluster. However, this cluster performs not destructive, but creative function. It “grows” in a series of harmonious chords that go along a separate line in parallel with the theme and sharp dissonant accords at the pianoforte. The next polyphonic episode, which grows as the instruments are connected and gradually captures an increasing range, leads to a bright “bell-like” culmination, which is replaced by a code that submerses the listener into a haze of uncertainty... What will happen next – who knows?..

In the course of discussing the concept of the work the question arose: what is dissonance? Is the interval of the increased octave (“F-natural” – “F-sharp”), which forms the basis of the conflict between harmony and chaos of this work, is an “absolute” dissonance? In the history of music, the answer to the question of consonances and dissonances constantly undergoes a rethinking (“discord today - consonance tomorrow”). It seems that the final verdict on the consonance or dissonance of a particular consonance can be made exclusively in this particular context. In the context of the same work, which contains a stylistic dialogue between two fundamentally different epochs (romanticism and modernity), the interval of the enlarged octave that breaks out from the harmonic environment is certainly perceived as a dissonance, a symbol of something alien to this world, a harbinger of chaos; whereas in the episode of the “ruins of the world” the same interval is no longer perceived as a dissonance, lives in its own environment and feels “at home”.

The question of what is dissonance and what consonances are the most pleasing to ears was of interest to Euler, who indirectly participates in this discussion. According to his theory, the interval of the increased octave in the natural scale appears between the 7th and 15th partial tones. For them

$$GS(7:15)=GS(3*5*7)=3+5+7-2=13.$$

This is a fairly large number. For comparison, a large seventh gives a smaller value:

$$GS(8:15)=GS(2*2*2*3*5)=2+2+2+3+5-4=10.$$

Consequently, after all, the enlarged octave is a dissonance, and, unfortunately, the world be left not leave without a dissonance; it is necessary to coexist with it, therefore, it is necessary to think musically, to take into account its presence. The question is, what will eventually outweigh – consonance equilibrium or dissonant tension...

Conclusions. The considered musical phenomena are innovative both in composition and in musicology; they reveal one of the interesting aspects of the “cooperation” of mathematics and music, when the object of the mathematical world becomes the starting point of the musical composition, penetrating into the structure of the musical fabric and transforming it in accordance with its needs. Transplanted into the sound sphere, this object generates a series of already completely musical thoughts of the author, develops further according to musical laws, but his symbolic fundamental principle continues to remind of himself throughout the entire musical development. The interaction of the introduced semantic stratum from the outside and collection of a priori internal musical meanings creates the effect of expanding the musical associative-conceptual complex, the discovery of new lingual possibilities of music.

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